Equalisation of Precession Rates of Galactic Orbits (Part 1)

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09 May 2013

Abstract

Using software which calculates precession rates of galactic orbits, a complete galactic component is constructed in the **f∝√d** field. It is proposed that interacting galactic orbits may adjust their ellipticities to equalise their precession rates.

The equalisation of precession rates within a structured component of a disk galaxy is essential if we wish to prevent the "winding problem" (1), in which the differential precession rates within a component would cause the structures we observe, such as galactic bars, and grand spirals of density waves, to be unsustainable in the long-term.

Systems of coplanar nested elliptic orbits are examined in a theoretical disk galaxy. Every orbit is assumed to be populated by a large number of stars.

The system of units used is chosen so that a circular orbit of radius r=1, around a point mass m=1, has orbital velocity $v_{circ}=1$, angular velocity $\omega_{circ}=1$, and orbital period $T_{circ}=2\pi$.
The axis ratio of an orbit is defined here as the minor axis divided by the major axis. The ellipticity is defined as 1 minus the axis ratio.

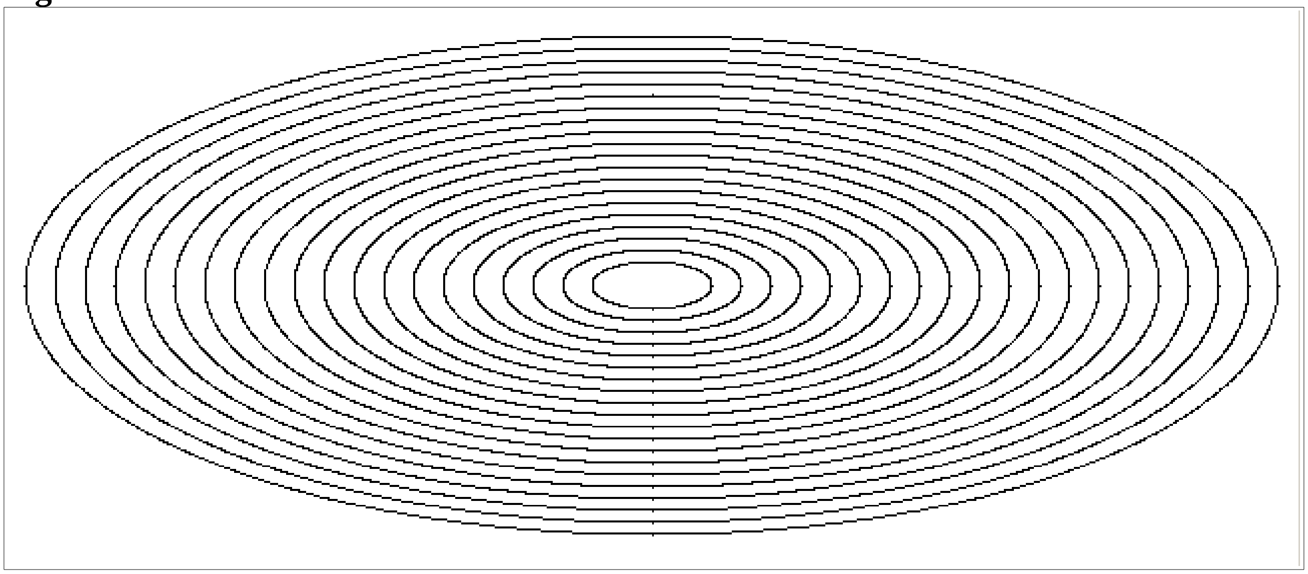
An essential part of this analysis is the ability to calculate the precession rate of orbits of various sizes and ellipticities, in a chosen galactic gravity field. This important problem has been examined extensively by others, for example in (2).

In the current work, an entirely empirical approach is chosen. A C++ orbit integration computer program was adapted to integrate galactic orbits in any chosen axisymmetric power-law galactic field. Additionally the program has the ability to automatically detect when a star is at apocentre, and to calculate empirically the azimuthal period, the radial period, the apsidal angle, and most importantly for the current analysis, the angular precession rate.

The first field examined is the theoretical axisymmetric galactic gravity field defined by **f∝√d**. The force attracting a unit mass towards the galactic centre is proportional to the **square root** of its distance from the galactic centre. Galactic orbits in this field are ellipses centred on the galactic centre and precess in the prograde direction.

First a system of nested orbits is examined which all have the same axis ratio = 0.4 as illustrated in figure 1.

Figure 1: Nested orbits all with axis ratio = 0.4



The precession rates of each of the orbits illustrated in figure 1 were calculated in the **f∝√d** field and are listed in figure 2.

Figure 2: Unequalised precession rates in the **f∝√d** field:

semi-major axis **a**	axis ratio **b/a**	precession rate $\boldsymbol{\omega_{prec}}$
0.10	0.4	0.1070
0.15	0.4	0.0964
0.20	0.4	0.0897
0.25	0.4	0.0849
0.30	0.4	0.0811
0.35	0.4	0.0780
0.40	0.4	0.0754
0.45	0.4	0.0733
0.50	0.4	0.0714
0.55	0.4	0.0697
0.60	0.4	0.0682
0.65	0.4	0.0668
0.70	0.4	0.0656
0.75	0.4	0.0645
0.80	0.4	0.0634
0.85	0.4	0.0625
0.90	0.4	0.0616
0.95	0.4	0.0608
1.00	0.4	0.0600
1.05	0.4	0.0593

The results listed in figure 2 demonstrate that in this field, we may examine any pair of adjacent orbits of equal axis ratio, and find that the smaller orbit precesses faster than the larger orbit. If two adjacent orbits are treated as rigid elliptical wires, then a huge torque must somehow be continuously applied, to keep the orbits with different precession rates aligned by brute force.

In this work, it is assumed instead, that each pair of adjacent nested orbits interacts specifically by altering the ellipticity of both orbits, and by this method equalises their precession rates. Then only tiny balancing torques are required to maintain the two adjacent orbits in a their state of equal precession.

A similar method has been applied previously by others, in solar system dynamics, to provide one of the several proposed explanations of how the eccentric planetary rings of Uranus and Saturn may exist. The analysis, by various authors (3), begins by considering that if all orbits within a given eccentric planetary ring have the same eccentricity, orbits of various sizes will precess at different rates, making the ring unsustainable. Then the possibility is examined that interactions between the orbits may modify their eccentricities and thereby may modify their precession rates. The resulting eccentricity gradient, which equalises the precession rates, is calculated. It is shown that, if this is the mechanism which equalises precession rates within an eccentric planetary ring, this would predict that all eccentric planetary rings will have a positive positive eccentricity gradient (eccentricity increasing as orbit size increases).

An example galactic component is constructed in the **f∝√d** field on the assumption that adjacent orbits interact by altering the ellipticities of the orbits, and thus equalise precession rates. The computer program does not model the interactions between adjacent orbits, instead the computer program is used to calculate the axis ratios of the orbits which would result from the assumed precession-equalising interactions.

First a single orbit is created, and the orbit is integrated by the computer program, which measures its axis ratio, and its precession rate. Next an orbit with a slightly greater semi-major axis is created, and its orbit is adjusted by iteration until its precession rate equals that of the first orbit, and the axis ratio of the orbit is recorded. This process is continued, by adding orbits of progressively larger semi-major axes, and adjusting the axis ratio of the added orbit until the precession rate is equal to that of the other orbits. The process is then repeated, by adding orbits with semi-major axes progressively smaller than the first orbit, and calculating the axis ratio of each orbit which equalises its precession rate to the precession rate of the first orbit. Thus a complete galactic component is constructed, within which every orbit has the same precession rate. The resulting orbits are calculated in the **f∝√d** field and presented in figure 3.

Figure 3: Equalised precession rates in the $\mathbf{f \propto \sqrt{d}}$ field:

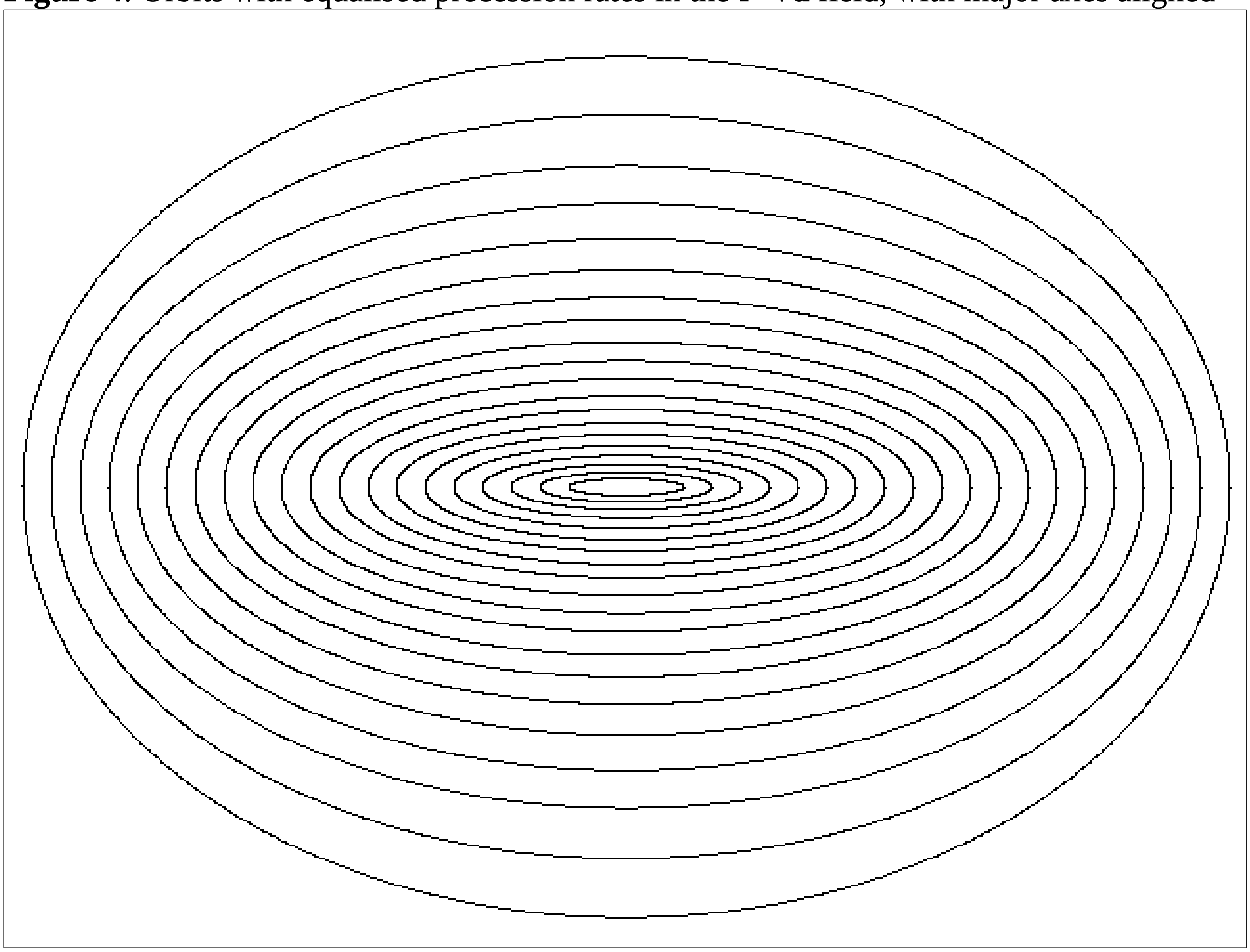
semi-major axis **a**	axis ratio **b/a**	precession rate $\boldsymbol{\omega_{prec}}$
0.001	0.032	≈0.066
0.01	0.065	≈0.066
0.05	0.113	≈0.066
0.10	0.146	≈0.066
0.15	0.174	≈0.066
0.20	0.196	≈0.066
0.25	0.217	≈0.066
0.30	0.241	≈0.066
0.35	0.263	≈0.066
0.40	0.279	≈0.066
0.45	0.299	≈0.066
0.50	0.316	≈0.066
0.55	0.343	≈0.066
0.60	0.367	≈0.066
0.65	0.389	≈0.066
0.70	0.419	≈0.066
0.75	0.442	≈0.066
0.80	0.473	≈0.066
0.85	0.509	≈0.066
0.90	0.549	≈0.066
0.95	0.590	≈0.066
1.00	0.649	≈0.066
1.05	0.715	≈0.066
1.10	0.794	≈0.065
1.15	0.858	≈0.063

The component has an outer limit, beyond which, if an attempt is made to add larger orbits, it becomes impossible for the larger orbits to achieve a precession rate equal to the precession rate of the existing component. For the two largest orbits in listed figure 3, it was found to be not possible, at any axis ratio, to achieve the component precession rate, and so the precession rates listed for these two large orbits do not match the precession rate of the component, but are the closest to a match that was achievable.

Progressively smaller orbits were added, and their precession rates matched to the component precession rate (pattern speed) by progressively increasing ellipticity, without the orbits becoming radial, see for example the three smallest orbits in figure 3.

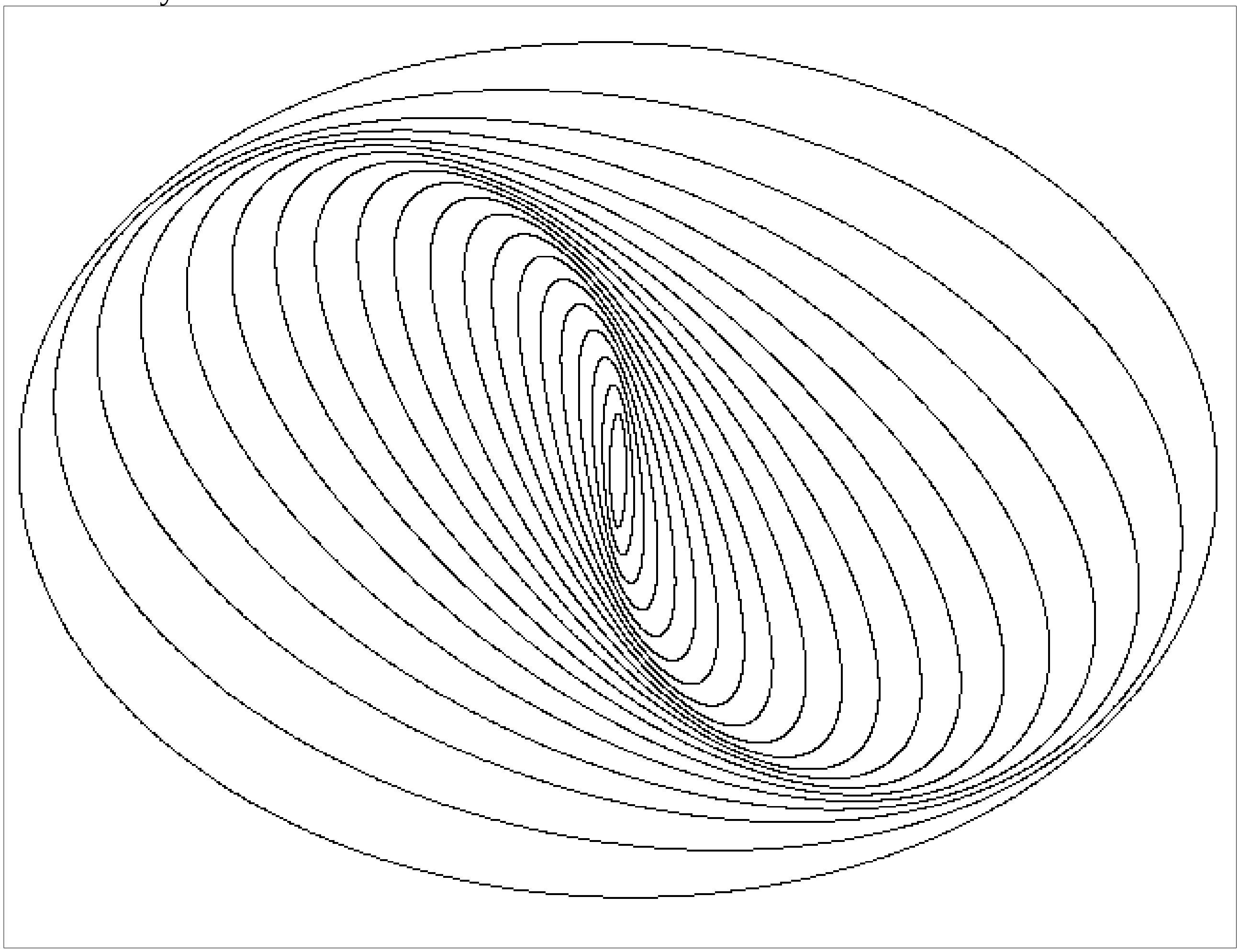
Twenty of the orbits listed in figure 3 are illustrated in figure 4. (The three smallest orbits and the two largest orbits are not illustrated). The orbits are in the clockwise direction. The direction of the apsidal precession of each orbit is clockwise (prograde). All the orbits have the same precession rate. The entire pattern of orbits precesses in the prograde direction.

Figure 4: Orbits with equalised precession rates in the **f∝√d** field, with major axes aligned



The question now arises, what relative alignments will the major axes of the orbits have, in the **f∝√d** field, when they are in the assumed precession-equalising interactive state? It seems likely that this may depend on various properties of the particular hypothetical **f∝√d** galaxy being examined, and that various relative alignments of major axes may be possible. Having illustrated one theoretical alignment in figure 4, another theoretical example is illustrated in figure 5, where it is assumed that the orbits achieve their stable interactive state with the major axes considerably rotated relative to each other. The orbits are clockwise. The entire pattern of orbits precesses in the prograde direction. The spiral kinematic density waves are trailing.

Figure 5: Orbits with equalised precession rates in the **f∝√d** field, with major axes differentially rotated.



The **f∝√d** field is scale-free, therefore the component may be constructed at any scale.

In this first part of the investigation, it has been proposed that the theoretical **f∝√d** field produces a galactic component with a defined outer limit, and with a defined ellipticity profile, in which orbital ellipticity decreases with increasing orbit size, and in which the uniform precession is prograde.

An HTML5 simulation of this set of nested orbits with uniform pattern speed is available (5). The simulator has the option to select a co-rotating viewing frame, which rotates at the precession rate, and which enables the co-precession of the numerous orbits to be clearly seen. It is the carefully tuned ellipticity gradient which produces the co-precession.

In the second part (4), it is intended to examine the very different $\mathbf{f \propto d^2}$ field, and to illustrate that it produces a galactic component rather different to that described above, in which orbital ellipticity increases with increasing orbit size, and in which the uniform precession is retrograde.

References

(1) Binney, J. & Tremaine, S.,
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(3) Murray, C., & Dermott, S.,
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(4) Equalisation of precession rates of galactic orbits (Part 2),
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(5) HTML5 simulation of the set of nested orbits with uniform pattern speed,
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